SOL HW 4.3

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Name: Key Date:

Section 4.3 Factoring Difference of Squares and Cubes

Difference of squares: $a^2 - b^2 = (a+b)(a-b)$

Difference and Sums of Cubes: $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$ $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

1. Factor each of the following expressions:

a) $16x^2 - 49y^2$ $(4x-7y)(4x+7y)$	b) $(2x-1)^2 - 9x^2$ $(2x-1)^2 - (3x)^2$ $[2x-1+3x][2x-1-3x]$ $=(5x-1)(-x-1) = -(5x-1)(x+1)$	c) $81a^2 - (3a+2b)^2$ $(9a)^2 - (3a+2b)^2$ $(9a+3a+2b)(9a-3a-2b)$ $(12a+2b)(6a-2b) = 4(6a+b)(3a-b)$
d) $4(2x-y)^2 - 25z^2$ $2^2(2x-y)^2 - 5^2z^2$ $=(2(2x-y)+5z)(2(2x-y)-5z)$ $=(4x-2y+5z)(4x-2y-5z)$	e) $18x^2y^2 - 50y^4$ $2[9x^2y^2 - 25y^4]$ $2(3xy+5y^2)(3xy-5y^2)$	f) $\frac{x^2}{16} - \frac{y^2}{49}$
g) $5x^4 - 80$ $5((x^2)^2 - (16))$ $5((x^2)^2 - 4^2)$ $5(x^2-4)(x^2+4)$ $5(x+2)(x-2)(x^2+4)$	h) $(3x-4)^3 + (x+3)^3$ $[3x-4+(x+3)][(3x-4)^2 - (3x-4)(x+3) + (x+3)^2]$ $[4x-1][9x^2-12x+16-(3x^2+5x-12)+x^2+6x+9]$ $(4x-1)[7x^2-11x+37]$	i) $(4x^2-4)^2 - 81x^4$ $(4x^2-4+9x^2)(4x^2-4-9x^2)$ $(13x^2-4)(-13x^2-4)$ $-(13x^2-4)(13x^2+4)$
j) $a^4 - 16a^6b^2$ $a^4(1-16a^2b^2)$ $a^4(1+4ab)(1-4ab)$	k) $a^6 + 7a^3 - 8$ $(a^3)^2 + 7a^3 - 8$ $(a^3-1)(a^3+8)$ $(a-1)(a^2+a+1)(a+2)(a^2-2a+4)$	L) $3a^4 - 15a^2 - 108$ $3\overline{)108}$ 36 $3(a^4-5a^2-36)$ $3(a^2-9)(a^2+4)$ $3(a+3)(a-3)(a^2+4)$
m) $27x^3 + 64y^3$ $(3x)^3 + (4y)^3$ $=(3x+4y)(9x^2 - 12xy + 16y^2)$	n) $125x^3 - 8x^3$ = $(5x)^3 - (2x)^3$ $(5x-2x)(25x^2 + 10x^2 + 4x^2)$	o) $250x^3 + 128y^3$ $2(125x^3 + 64y^3)$ $2(5x+4y)(25x^2 - 20xy + 16y^2)$

2. For what value of "n" does $(2^{2007} - 2^{2006})(2^{1997} - 2^{1996}) = 2^n$?

$2^4 - 2^3 = 2^3 \{ (2^4 - 2^3)$, $2^3(2-1)$, 2^3

$2^5 - 2^4 = 2^4$

$2^{2006}(2-1)\,2^{1996}(2-1) = 2^n$

$2^{2006}(1)\,2^{1996}(1) = 2^n$

$2^{4002} = 2^n$

$\boxed{4002 = n}$

3. The number 2001 can be written as a difference of squares, $x^2 - y^2$ where "x" and "y" are positive integers in four different ways. What are the four possible ways?

$(x+y)(x-y) = 2001$
$= 2001 \times 1$ ①
$= 667 \times 3$ ②
$= 87 \times 23$ ③
$= 69 \times 29$ ④

① $x+y=2001$, $x-y=1$, $2x=2002$, $\boxed{x=1001,\ y=1000}$

② $x+y=667$, $x-y=3$, $2x=670$, $\boxed{x=335,\ y=332}$

③ $x+y=87$, $x-y=23$, $2x=110$, $\boxed{x=55,\ y=32}$

④ $x+y=69$, $x-y=29$, $2x=98$, $\boxed{x=49,\ y=20}$

4. Two numbers are such that their difference, their sum and their product are to one another as 1 : 7 : 18. The product of the two numbers are:

a) 6 b) 12 c) 24 d) 48 $\boxed{\text{e) none of these}}$

① $a-b,\ a+b,\ a\times b$; $k,\ 7k,\ 18k$.
$a-b=k$, $a+b=7k$, $2a=8k$, $\boxed{a=4k}$; $a\times b=18k$, $(4k)b=18k$, $\boxed{b=4.5}$; $\therefore a=$

② $a+b=7k$, $a-b=1k$, $2b=6k$, $2(4.5)=6k$, $9=6k$, $\frac{3}{2}=k$, $a=4(\frac{3}{2})$, $\boxed{a=6}$

③ $6-4.5,\ 6+4.5,\ 6\times4.5$; $1.5,\ 10.5,\ 27.$

5. The number 2005 can be written in the form of $a^2 - b^2$, where "a" and "b" are positive integers less than 1000 in exactly one way. What is the value of $a^2 + b^2$?

① $2005 = a^2-b^2$
$2005 = (a+b)(a-b)$
401×5
$a+b=401$
$a-b=5$
$2a=406$
$a=203$, $b=198$

② a^2+b^2
$203^2+198^2=$
$41209+39204$
$=80413$

6. Solve for "a" and "b" in the expression: $1+\sqrt{8} = \left(a+\sqrt{2}\right)\left(b-\sqrt{2}\right)$

$ab - a\sqrt{2} + b\sqrt{2} - 2 = 1 + 2\sqrt{2}$

$ab - 2 + (b-a)\sqrt{2} = 1 + 2\sqrt{2}$

$ab-2=1$
$\boxed{ab=3}$
$a=\frac{3}{b}$

$b-a=2$
$b-\frac{3}{b}=2$
$b^2-2b-3=0$

7. Given that "p" is a prime number, solve for "x": $1000 - 8x^3 = 8(p)(5p+2)$

$8(125) - 8x^3 = 8p(5p+2)$

$8(5^3 - x^3) = 8p(5p+2)$

$8(5-x)\boxed{(25+5x+x^2)} = 8p(5p+2)$

8. Suppose that $n^2 - 4 = 50(n-2)$ and "n" is not equal to 2. What is the value of "n"?

$(n+2)(n-2) = 50(n-2)$
$n+2=50$
$\boxed{n=48}$

9. Find as many prime numbers "p" as you can so that the expression 5p+1 is a perfect square. How many prime numbers like this do you think there are? Prove that there are only this many primes.

$5p+1=x^2$ (SAME QUESTION AS LESSON ONLINE)

$5p=(x+1)(x-1)$

10. The positive difference of two perfect squares is 32. What is the largest possible value of the sum of the two perfect squares?

$x^2-y^2=32$

$(x+y)(x-y)=32$

32 1 ①
16 2 ②
8 4 ③

① $x+y=32$, $x-y=1$, $2x=33$, $x=\frac{33}{2}$, $y=\frac{31}{2}$ (NOT INTEGERS, NO PERFECT SQUARES)

② $x+y=16$, $x-y=2$, $2x=18$, $x=9$, $y=7$; x^2+y^2, 9^2+7^2, $81+49=130$

③ $x+y=8$, $x-y=4$, $2x=12$, $x=6$, $y=2$; x^2+y^2, $36+4=40$

LARGEST IS 130

11. How many ordered pairs (m,n) of $\boxed{\text{positive integers}}$, with $\boxed{m>n}$, have the property that their squares differ by 96? List out all possible pairs.

$m^2-n^2=96$

$(m+n)(m-n)=96$

96 × 1 ①
48 × 2 ②
32 × 3 ③
24 × 4 ④
16 × 6 ⑤
12 × 8 ⑥

① $m+n=96$, $m-n=1$, $2m=97$, $m=\frac{97}{2}$ (NO)

② $m+n=48$, $m-n=2$, $2m=50$, $\boxed{m=25}$, $\boxed{n=23}$

③ $m+n=32$, $m-n=3$, $2m=35$ (NO)

④ $(m+n)=24$, $m-n=4$, $2m=28$, $\boxed{m=14}$, $\boxed{n=10}$

⑤ $m+n=16$, $m+n=6$, $2m=22$, $\boxed{m=11}$, $\boxed{n=5}$

⑥ $m+n=12$, $m-n=8$, $2m=20$, $\boxed{m=10}$, $\boxed{n=2}$

4 pairs (x,y): (25,23), (14,10), (11,5) (10,2).

12. Given that "p" is a prime number and the expression 2003p + 16 is a perfect square, what is the lowest possible value of "p"? (hint:)

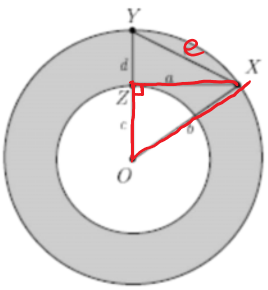
① $2003p+16=k^2$

$2003p=k^2-16$

$2003p=(k+4)(k-4)$

13. An annulus is the region between two concentric circles. The concentric circles in the figure have radii "b" and "c", with b > c. Let $\overline{OX}$ be a radius of the larger circle, let $\overline{XZ}$ be tangent to the smaller circle at Z, and let $\overline{OY}$ be the radius of the larger circle that contains Z. Let *a* = *XZ*, *d* = *YZ*, and *e* = *XY*. What is the area of the annulus?

(A) πa^2 **(B)** πb^2 **(C)** πc^2 **(D)** πd^2 **(E)** πe^2



$\pi b^2-\pi c^2$

$\pi(b^2-c^2)$

$\pi(a^2)$

14. If $a^2-b^2=64$, what is the smallest possible values of $(a+b)$?

$(a+b)(a-b)=64$

1 × 64
2 × 32
4 × 16
8 × 8
$\boxed{-64 \times -1}$
-32×-2

SINCE (a,b) CAN BE NEGATIVE, THEN THE SMALLEST VALUE WOULD BE −64

15. There are four different positive integers "a", "b", "c" , and "d" such that the equation is true:

$a^3 + b^3 = c^3 + d^3 = 1729$. What is the values of *a* + *b* + *c* + *d*?

$\boxed{1^3 = 1}$ $6^3 = 216$ $11^3 = 1331$
$2^3 = 8$ $7^3 = 343$ $\boxed{12^3 = 1728}$
$3^3 = 27$ $8^3 = 512$
$4^3 = 64$ $9^3 = 729$ } c
$5^3 = 125$ $10^3 = 1000$ } d

$\boxed{a = 1,\ b = 12,\ c = 9,\ d = 10}$

16. What is the sum of the (decimal) digits of $10^6 - 8$?

1 000 000 − 8 = 999992

17. What are both pairs of integers (x,y) for which $4^y - 615 = x^2$

$2^{2y} - x^2 = 615$

$(2^y + x)(2^y - x) = 615$

1	615 → 616x
5	123 → 128 ✓
3	205 → 208x
41	15 → 56x
615	1x
123	5 ✓
205	3x
15	41x

$2^y + \cancel{x} + 2^y - \cancel{x} = 2 \cdot 2^y = 2^{y+1}$

$2^{y+1} = 128$
$2^{y+1} = 2^7$
$y + 1 = 7$
$\boxed{y = 6}$

$\boxed{2^y + x = 5 \quad 2^y - x = 123}$
64 − 59 = 5 or

$2^y + x = 123 \quad 2^y - x = 5$
64 + 59 = 123

(−59, 6)
or
(59, 6)

18. Show that if the positive integer "n" is a multiple of 3, then $7^n - 6^n$ is a multiple of 127.

① $\boxed{a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \cdots b^{n-1})}$

$2^7 - b^7 = (2-b)(2^6 + 2^5 b + 2^4 b^2 + 2^3 b^3 + 2^2 b^4 + 2^1 b^5 + b^6)$

② $a^n + b^n = (a+b)(a^{n-1} - a^{n-2}b + a^{n-3}b^2 - a^{n-4}b^3 + \cdots + b^{n-1})$

$7^{3k} - 6^{3k} = (7^3)^k - (6^3)^k = (7^3 - 6^3)[(7^3)^{k-1} + (7^3)^{k-2}(6^3) + (7^3)^{k-2}(6^3)^2 \cdots]$

343 − 216

127 [. . . . .]

20 Show that $2^{99} + 3^{99}$ is divisible by 35.

$(2^3)^{33} + (3^3)^{33}$

$= [2^3 + 3^3][(2^3)^{32} + (2^3)^{31}(3^3) + \cdots]$

8 + 27

21 (Challenge) Let "x" and "y" be two-digit integers such that "y" is obtained by reversing the digits of "x". The integers "x" and "y" satisfy $x^2 - y^2 = m^2$ for some positive integer "m". What is the value of $x + y + m$?

Hint:
LET = AB = 10A+B = x.
Then BA = 10B + A = y.

So: $x^2 - y^2 = m^2$
$(10A + B)^2 - (10B + A)^2 = m^2$ ← From this step on, manipulate the eqn to see what you can get.